

Problem 9

Use equation (1.8) to find the fractions that are equivalent to the following repeating decimals:

$$0.857142857142\dots$$

Solution

$$\begin{aligned}0.857142857142\dots &= 0.857142 + 0.000000857142 + \dots \\&= \frac{857\,142}{10^6} + \frac{857\,142}{10^{12}} + \dots \\&= \sum_{i=1}^{\infty} \frac{857\,142}{10^{6i}} \\&= \sum_{i=1}^{\infty} \frac{857\,142}{1\,000\,000^i} \\&= 857\,142 \sum_{i=1}^{\infty} \frac{1}{1\,000\,000^i} \\&= 857\,142 \sum_{i=1}^{\infty} \left(\frac{1}{1\,000\,000}\right)^i \\&= 857\,142 \left[-1 + \sum_{i=0}^{\infty} \left(\frac{1}{1\,000\,000}\right)^i\right] \\&= 857\,142 \left[-1 + \frac{1}{1 - \left(\frac{1}{1\,000\,000}\right)}\right] \\&= 857\,142 \left(-1 + \frac{1\,000\,000}{999\,999}\right) \\&= 857\,142 \left(\frac{1}{999\,999}\right) \\&= \frac{6}{7}\end{aligned}$$